

Parton distribution functions and quark orbital motion *

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Covariant version of the quark-parton model is studied. Dependence of the structure functions and parton distributions on the 3D quark intrinsic motion is discussed. The important role of the quark orbital momentum, which is a particular case of intrinsic motion, appears as a direct consequence of the covariant description. Effect of orbital motion is substantial especially for polarized structure functions. At the same time, the procedure for obtaining the quark momentum distributions of polarized quarks from the combination of polarized and unpolarized structure functions is suggested.

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1. INTRODUCTION

The nucleon structure functions are basic tool for understanding the nucleon internal structure in the language of QCD. And at the same time, the measuring and analysis of the structure functions represent the important experimental test of this theory. Unpolarized nucleon structure functions are known with high accuracy in very broad kinematical region, but in recent years also some precision measurements on the polarized structure functions have been completed [1, 2, 3, 4, 5, 6, 7]. For present status of the nucleon spin structure see e.g. [8] and citations therein. The more formal aspects of the nucleon structure functions are explained in [9]. In fact only the complete set of the four electromagnetic unpolarized and polarized structure functions F_1, F_2, g_1 and g_2 can give a consistent picture of the nucleon. However, this picture is usually drawn in terms of the distribution functions, which are connected with the structure functions by some model-dependent way. Distribution functions are not directly accessible from the experiment and model, which is normally applied for their extraction from the structure functions is the well known quark-parton model (QPM). Application of this model for analysis and interpretation of the unpolarized data does not create any contradiction. On the other hand, the situation is much less clear in the case of spin functions g_1 and g_2 .

In our previous study [11, 12] we have suggested, that a reasonable explanation of the experimentally measured spin functions g_1, g_2 is possible in terms of a generalized covariant QPM, in which the quark intrinsic motion (i.e. 3D motion with respect to the nucleon rest frame) is consistently taken into account. Therefore the quark transversal momentum appears in this approach on the same level as the longitudinal one. The quarks are represented by the free Dirac spinors, which allows to obtain exact and covariant solution for relations between the quark momentum distribution functions and the structure functions accessible from experiment. In this way the model (in its present LO version) contains no dynamics but only “exact” kinematics of quarks, so it can be effective tool for analysis and interpretation of the experimental data on structure functions, particularly for separating effects of the dynamics (QCD) from effects of the kinematics. This point of view is well supported by our previous results:

a) In the cited papers we showed, that the model simply implies the well known sum rules (Wanzura-Wilczek, Efremov-Leader-Teryaev, Burkhardt-Cottingham) for the spin functions g_1, g_2 .

b) Simultaneously, we showed that the same set of assumptions implies rather substantial dependence of the first moment Γ_1 of the function g_1 on the kinematical effects.

c) Further, we showed that the model allows to calculate the functions g_1, g_2 from the unpolarized valence quark distributions and the result is quite compatible with the experimental data.

d) In the paper [13] we showed that the model allows to relate the transversity distribution to some other structure functions.

These results cannot be obtained from the standard versions of the QPM (naive or the QCD improved), which are currently used for the analysis of experimental data on structure functions. The reason is, that the standard QPM is based on the simplified and non-covariant kinematics in the infinite momentum frame (IMF), which does not allow to properly take into account the quark intrinsic or orbital motion.

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The subject of our previous study was the question: *What is the dependence of the structure functions on quark intrinsic motion?* The aim of the present paper is a discussion of related problems:

1. *How to extract information about the quark intrinsic motion from the experimentally measured structure functions?*

2. *What is the role of the quark orbital momentum, which is a particular case of intrinsic motion?*

The paper is organized as follows. In the first part of Sec. 2 the basic formulas, which follow from the generalized QPM, are presented. Resulting general covariant relations are compared with their limiting case, which is represented by the standard formulation of the QPM in the IMF. In the next part of the section the relations for calculation of 3D quark momentum distributions from the structure functions are derived. The quark momentum distributions of positively and negatively polarized quarks are separately obtained from the combination of structure functions F_2, g_1 or corresponding parton distributions $q(x), \Delta q(x)$. The particular form of the quark intrinsic motion is the orbital momentum. In Sec. 3 the role of the quark orbital momentum in covariant description is discussed and it is shown, why its contribution to the total quark angular momentum can be quite substantial. It is demonstrated, that the orbital motion is an inseparable part of the covariant approach. The last section is devoted to a short summary and conclusion. In fact, this paper is inspired by many previous papers, see e.g. [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24], in which problem of quark orbital momentum in the context of nucleon spin was recognized and studied.

2. STRUCTURE FUNCTIONS AND INTRINSIC QUARK MOTION

In our previous study [10, 11, 12] of the proton structure functions we showed, how these functions depend on the intrinsic motion of quarks. The quarks in the suggested model are represented by the free fermions, which are in the nucleon rest frame described by the set of distribution functions with spheric symmetry $G_k^\pm(p_0)d^3p$, where $p_0 = \sqrt{m^2 + \mathbf{p}^2}$ and symbol k represents the quark and antiquark flavors. These distributions measure the probability to find a quark of given flavor in the state

$$u(\mathbf{p}, \lambda \mathbf{n}) = \frac{1}{\sqrt{N}} \left(\frac{\phi_{\lambda \mathbf{n}}}{\frac{\mathbf{p} \cdot \boldsymbol{\sigma}}{p_0 + m} \phi_{\lambda \mathbf{n}}} \right); \quad \frac{1}{2} \mathbf{n} \cdot \boldsymbol{\sigma} \phi_{\lambda \mathbf{n}} = \lambda \phi_{\lambda \mathbf{n}}, \quad N = \frac{2p_0}{p_0 + m}, \quad (1)$$

where m and p are the quark mass and momentum, $\lambda = \pm 1/2$, $\phi_{\lambda \mathbf{n}}^\dagger \phi_{\lambda \mathbf{n}} = 1$ and \mathbf{n} coincides with the direction of nucleon polarization. The distributions with the corresponding quark (and antiquark) charges e_k allow to define the generic functions G and ΔG^1 ,

$$G(p_0) = \sum_k e_k^2 G_k(p_0), \quad G_k(p_0) \equiv G_k^+(p_0) + G_k^-(p_0), \quad (2)$$

$$\Delta G(p_0) = \sum_k e_k^2 \Delta G_k(p_0), \quad \Delta G_k(p_0) \equiv G_k^+(p_0) - G_k^-(p_0), \quad (3)$$

from which the structure functions can be obtained. If q is momentum of the photon absorbed by the nucleon of the momentum P and mass M , in which the phase space of quarks is controlled by the distributions $G_k^\pm(p_0)d^3p$, then there are the following representations of corresponding LO structure functions.

A. *Manifestly covariant representation*

i) *unpolarized structure functions:*

$$F_1(x) = \frac{M}{2} \left(A + \frac{B}{\gamma} \right), \quad F_2(x) = \frac{Pq}{2M\gamma} \left(A + \frac{3B}{\gamma} \right), \quad (4)$$

where

$$A = \frac{1}{Pq} \int G \left(\frac{Pp}{M} \right) [pq - m^2] \delta \left(\frac{pq}{Pq} - x \right) \frac{d^3p}{p_0}, \quad (5)$$

¹ In the papers [11, 12] we used different notation for the distributions defined by Eqs.(2) and (3): $G_k^\pm, \Delta G_k$ and ΔG were denoted as $h_{k\pm}, \Delta h_k$ and H . Apart of that we assumed for simplicity that only three (valence) quarks contribute to the sums (2) and (3). In present paper we assume contribution of all the quarks and antiquarks, but apparently general form of the relations like (10) - (12) is in the LO approach independent of chosen set of quarks.

$$B = \frac{1}{Pq} \int G \left(\frac{pP}{M} \right) \left[\left(\frac{Pp}{M} \right)^2 + \frac{(Pp)(Pq)}{M^2} - \frac{pq}{2} \right] \delta \left(\frac{pq}{Pq} - x \right) \frac{d^3p}{p_0} \quad (6)$$

and

$$\gamma = 1 - \left(\frac{Pq}{Mq} \right)^2. \quad (7)$$

The functions $F_1 = MW_1$ and $F_2 = (Pq/M)W_2$ follow from the tensor equation

$$\begin{aligned} & \left(-g_{\alpha\beta} + \frac{q_\alpha q_\beta}{q^2} \right) W_1 + \left(P_\alpha - \frac{Pq}{q^2} q_\alpha \right) \left(P_\beta - \frac{Pq}{q^2} q_\beta \right) \frac{W_2}{M^2} \\ &= \int G \left(\frac{pP}{M} \right) [2p_\alpha p_\beta + p_\alpha q_\beta + q_\alpha p_\beta - g_{\alpha\beta} pq] \delta \left((p+q)^2 - m^2 \right) \frac{d^3p}{p_0}. \end{aligned} \quad (8)$$

By modification of the delta function term

$$\delta \left((p+q)^2 - m^2 \right) = \delta (2pq + q^2) = \delta \left(2Pq \left(\frac{pq}{Pq} - \frac{Q^2}{2Pq} \right) \right) = \frac{1}{2Pq} \delta \left(\frac{pq}{Pq} - x \right); \quad q^2 = -Q^2, \quad x = \frac{Q^2}{2Pq}, \quad (9)$$

the dependence on the Bjorken x is introduced. Then contracting with the tensors $g_{\alpha\beta}$ and $P_\alpha P_\beta$ gives the set of two equations, which determine the functions F_1, F_2 in accordance with Eqs. (4)-(7).

ii) polarized structure functions:

As follows from [11] the corresponding spin functions in covariant form read

$$g_1 = Pq \left(G_S - \frac{Pq}{qS} G_P \right), \quad g_2 = \frac{(Pq)^2}{qS} G_P, \quad (10)$$

where S is the nucleon spin polarization vector and the functions G_P, G_S are defined as

$$G_P = \frac{m}{2Pq} \int \Delta G \left(\frac{pP}{M} \right) \frac{pS}{pP + mM} \left[1 + \frac{1}{mM} \left(pP - \frac{pu}{qu} Pq \right) \right] \delta \left(\frac{pq}{Pq} - x \right) \frac{d^3p}{p_0}, \quad (11)$$

$$G_S = \frac{m}{2Pq} \int \Delta G \left(\frac{pP}{M} \right) \left[1 + \frac{pS}{pP + mM} \frac{M}{m} \left(pS - \frac{pu}{qu} qS \right) \right] \delta \left(\frac{pq}{Pq} - x \right) \frac{d^3p}{p_0}; \quad (12)$$

$$u = q + (qS)S - \frac{(Pq)}{M^2}P.$$

B. Rest frame representation for $Q^2 \gg 4M^2x^2$

As follows from the Appendix in [11], if $Q^2 \gg 4M^2x^2$ and the above integrals are expressed in terms of the nucleon rest frame variables, then one can substitute

$$\frac{pq}{Pq} \rightarrow \frac{p_0 + p_1}{M}$$

and the structure functions are simplified as:

$$F_1(x) = \frac{Mx}{2} \int G(p_0) \delta \left(\frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}, \quad (13)$$

$$F_2(x) = Mx^2 \int G(p_0) \delta \left(\frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}, \quad (14)$$

$$g_1(x) = \frac{1}{2} \int \Delta G(p_0) \left(m + p_1 + \frac{p_1^2}{p_0 + m} \right) \delta \left(\frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}, \quad (15)$$

$$g_2(x) = -\frac{1}{2} \int \Delta G(p_0) \left(p_1 + \frac{p_1^2 - p_T^2/2}{p_0 + m} \right) \delta \left(\frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}, \quad (16)$$

where the p_1 and p_T are longitudinal and transversal quark momentum components. These structure functions consist of terms like

$$q(x) = Mx \int G_q(p_0) \delta\left(\frac{p_0 + p_1}{M} - x\right) \frac{d^3p}{p_0}, \quad (17)$$

$$\Delta q(x) = \int \Delta G_q(p_0) \left(m + p_1 + \frac{p_1^2}{p_0 + m}\right) \delta\left(\frac{p_0 + p_1}{M} - x\right) \frac{d^3p}{p_0}, \quad (18)$$

which correspond to the contributions from different quark flavors, $q = u, \bar{u}, d, \bar{d}, s, \bar{s}, \dots$. Let us remark, in limit of the IMF approach (see next paragraph) these functions represent probabilistic distributions of the quark momentum in terms of the fraction x of the nucleon momentum, $p = xP$. However content and interpretation of the functions (17),(18) depending on the Bjorken x is more complex; their form reflects in a non-trivial way intrinsic 3D motion of quarks.

C. Standard IMF representation

The usual formulation of the QPM gives the known relations between the structure functions and the parton distribution functions [9]:

$$F_1(x) = \frac{1}{2} \sum_q e_q^2 q(x), \quad F_2(x) = x \sum_q e_q^2 q(x), \quad (19)$$

$$g_1(x) = \frac{1}{2} \sum_q e_q^2 \Delta q(x), \quad g_2(x) = 0, \quad (20)$$

where the functions

$$q(x) = q^+(x) + q^-(x), \quad \Delta q(x) = q^+(x) - q^-(x) \quad (21)$$

represent probabilistic distributions of the of the quark momentum fraction x in the IMF. In the Appendix A we have proved that these relations represent the particular, limiting case of the covariant relations (4) and (10).

The three versions of the relations between the structure functions and the quark distributions can be compared:

a) If we skip the function g_2 in the version *C*, then the relations (19) and (20) practically represent identity between the structure functions and distributions of the quark momentum fraction. Such simple relations are valid only for the IMF approach based on the approximation (A1), which means that the quark intrinsic motion is suppressed. In more general versions *A* and *B*, where the intrinsic motion is allowed, the relations are more complex. The intrinsic motion strongly modifies also the g_2 . In the version *C* there is $g_2(x) = 0$, but $g_2(x) \neq 0$ in the *A* and *B*.

b) The version *B* allows to easily calculate the (substantial) dependence of the first moment Γ_1 on the rate of intrinsic motion. A more detailed discussion follows in the next section. The same approach implies that functions g_1 and g_2 for massless quarks satisfy the relation equivalent to the Wandzura-Wilczek term and obey some well known sum rules, that is shown in [11].

c) The functions F_1 and F_2 exactly satisfy the Callan-Gross relation $F_2(x)/F_1(x) = 2x$ in the versions *B* and *C*, but this relation is satisfied only approximately in the *A*: $F_2(x)/F_1(x) \approx 2x + O(4M^2x^2/Q^2)$.

The task which was solved in different approximations above can be formulated: How to obtain the structure functions F_1, F_2 and g_1, g_2 from the probabilistic distributions G and ΔG defined by Eqs. (2) and (3)? In the next we will study the inverse problem, the aim is to find out a rule for obtaining the distribution functions G and ΔG from the structure functions. In the present paper we consider the functions F_2 and g_1 represented by Eqs. (14) and (15). As follows from the Appendix A in [12], the function

$$V_n(x) = \int K(p_0) \left(\frac{p_0}{M}\right)^n \delta\left(\frac{p_0 + p_1}{M} - x\right) d^3p \quad (22)$$

satisfies

$$V'_n(x_{\pm})x_{\pm} = \mp 2\pi M K(\xi) \xi \sqrt{\xi^2 - m^2} \left(\frac{\xi}{M}\right)^n; \quad x_{\pm} = \frac{\xi \pm \sqrt{\xi^2 - m^2}}{M}. \quad (23)$$

In this section we consider only the case $m \rightarrow 0$, then

$$V'_n(x)x = -2\pi M K(\xi) \xi^2 \left(\frac{\xi}{M}\right)^n; \quad x = \frac{2\xi}{M}. \quad (24)$$

As we shall see below, with the use of this relation one can obtain the probabilistic distributions $G(p)$ and $\Delta G(p)$ from the experimentally measured structure functions. The same procedure will be applied to get the $G_q(p)$ and $\Delta G_q(p)$ from the usual parton distributions $q(x)$ and $\Delta q(x)$ defined by Eqs. (19) - (20).

Let us remark that in present stage the QCD evolution is not included into the model. However, this fact does not represent any restriction for the present purpose - to obtain information about distributions of quarks at some scale Q^2 from the structure functions measured at the same Q^2 . Distribution of the gluons is another part of the nucleon picture. But since our present discussion is directed to the relation between the structure functions and corresponding (quark) distributions at given scale, the gluon distribution is left aside.

2.1. Momentum distribution from structure function F_2

In an accordance with the definition (22) in which the distribution $K(p_0)$ is substituted by the $G(p_0)$, the structure function (14) can be written in the form

$$F_2(x) = x^2 V_{-1}(x). \quad (25)$$

Then, with the use of the relation (24) one gets

$$G(p) = -\frac{1}{\pi M^3} \left(\frac{F_2(x)}{x^2} \right)' = \frac{1}{\pi M^3 x^2} \left(\frac{2F_2(x)}{x} - F_2'(x) \right); \quad x = \frac{2p}{M}, \quad p \equiv \sqrt{\mathbf{p}^2} = p_0, \quad (26)$$

which in terms of the quark distributions means

$$G_q(p) = -\frac{1}{\pi M^3} \left(\frac{q(x)}{x} \right)' = \frac{1}{\pi M^3 x^2} (q(x) - xq'(x)). \quad (27)$$

Probability distribution G_q measures number of quarks of flavor q in the element d^3p . Since $d^3p = 4\pi p^2 dp$, the distribution measuring the number of quarks in the element dp/M reads:

$$P_q(p) = 4\pi p^2 M G_q(p) = -x^2 \left(\frac{q(x)}{x} \right)' = q(x) - xq'(x). \quad (28)$$

The probability distribution $G_q(p)$ is positive, so the last relation implies

$$\left(\frac{q(x)}{x} \right)' \leq 0. \quad (29)$$

Let us note, the maximum value of quark momentum is $p_{\max} = M/2$, which is a consequence of the kinematics in the nucleon rest frame, where the single quark momentum must be compensated by the momentum of the other partons.

Another quantity, which can be obtained, is the distribution of the quark transversal momentum. Obviously the integral

$$\frac{dN_q}{dp_T^2} = \int G_q(p) \delta(p_2^2 + p_3^2 - p_T^2) d^3p, \quad (30)$$

which represents the number of quarks in the element dp_T^2 , can be modified as

$$\frac{dN_q}{dp_T^2} = 2\pi \int_0^{\sqrt{p_{\max}^2 - p_T^2}} G_q \left(\sqrt{p_1^2 + p_T^2} \right) dp_1. \quad (31)$$

It follows, that the distribution corresponding to the number of quarks in the element dp_T/M reads:

$$P_q(p_T) = M \frac{dN_q}{dp_T} = 4\pi p_T M \int_0^{\sqrt{p_{\max}^2 - p_T^2}} G_q \left(\sqrt{p_1^2 + p_T^2} \right) dp_1. \quad (32)$$

Then with the use of Eq. (28) one gets the distribution

$$P_q(p_T) = \frac{4p_T}{M^2} \int_0^{\sqrt{p_{\max}^2 - p_T^2}} \frac{1}{x^2} (q(x) - xq'(x)) dp_1; \quad x = \frac{2\sqrt{p_1^2 + p_T^2}}{M}. \quad (33)$$

In Fig. 1 the distributions (28) and (33) are displayed for the valence and sea quarks. As an input we used the standard parameterization [25] of the parton distribution functions $q(x)$, $\bar{q}(x)$ (LO at the scale 4GeV^2). Resulting distributions $P_q, P_{\bar{q}}$ are positive, it means that the input distributions q, \bar{q} satisfy the constraint (29).

Using the Eq. (27) one can calculate the mean values

$$\langle p \rangle_q = \frac{\int p G_q(p) d^3 p}{\int G_q(p) d^3 p} = \frac{M}{2} \frac{\int_0^1 x (q(x) - x q'(x)) dx}{\int_0^1 (q(x) - x q'(x)) dx}. \quad (34)$$

In the case of sea quarks extrapolation of the distribution functions for $x \rightarrow 0$ gives a divergent integral in the denominator, it follows that $\langle p \rangle_{sea} \rightarrow 0$. For the valence quarks $q_{val} = q - \bar{q}$ this integral converges and integration by parts gives

$$\langle p \rangle_{q, val} = \frac{3M}{4} \frac{\int_0^1 x q_{val}(x) dx}{\int_0^1 q_{val}(x) dx}. \quad (35)$$

Calculation $\langle p \rangle_{q, val}$ gives roughly $0.11\text{GeV}/c$ for u and $0.083\text{GeV}/c$ for d quarks. Since $G_q(p)$ has rotational symmetry, average transversal momentum can be calculated as $\langle p_T \rangle = \pi/4 \cdot \langle p \rangle$.

2.2. Momentum distribution from structure function g_1

In the paper [12], Eq. (44), we proved that

$$g_1(x) = V_0(x) - \int_x^1 \left(4 \frac{x^2}{y^3} - \frac{x}{y^2} \right) V_0(y) dy, \quad (36)$$

where the function V_0 is defined by Eq. (22) for $n = 0$ and $K(p) = \Delta G(p)$. In the Appendix B it is shown, that the last relation can be modified to:

$$V_{-1}(x) = \frac{2}{x} \left(g_1(x) + 2 \int_x^1 \frac{g_1(y)}{y} dy \right). \quad (37)$$

Then, in an accordance with Eq. (24), we obtain

$$V'_{-1}(x) = -\pi M^3 \Delta G(p); \quad x = \frac{2p}{M}, \quad (38)$$

so the last two relations imply

$$\Delta G(p) = -\frac{2}{\pi M^3} \left[\frac{1}{x} \left(g_1(x) + 2 \int_x^1 \frac{g_1(y)}{y} dy \right) \right]'; \quad x = \frac{2p}{M}, \quad (39)$$

or

$$\Delta G(p) = \frac{2}{\pi M^3 x^2} \left(3g_1(x) + 2 \int_x^1 \frac{g_1(y)}{y} dy - x g_1'(x) \right); \quad x = \frac{2p}{M}. \quad (40)$$

Now we substitute

$$\Delta q(x) = g_1(x)/2, \quad g_1(x) + g_2(x) = \int_x^1 \frac{g_1(y)}{y} dy = \Delta q_T(x)/2 \quad (41)$$

and in the next we shall consider the flavors separately. The second equality represents Wanzura-Wilczek relation for twist-2 approximation of g_2 , which is valid for present approach as proved in [12]. Now Eq. (40) in terms of the quark distributions reads

$$\Delta G_q(p) = \frac{1}{\pi M^3 x^2} \left(3\Delta q(x) + 2 \int_x^1 \frac{\Delta q(y)}{y} dy - x \Delta q'(x) \right); \quad x = \frac{2p}{M} \quad (42)$$

or equivalently, with the use of Eqs. (39) and (41):

$$\Delta G_q(p) = -\frac{1}{\pi M^3} \left(\frac{\Delta q(x) + 2\Delta q_T(x)}{x} \right)'; \quad x = \frac{2p}{M}. \quad (43)$$

Obviously the distribution ΔG_q together with the distribution (27) allow to obtain the polarized distributions G_q^\pm as

$$G_q^\pm(p) = \frac{1}{2} (G_q(p) \pm \Delta G_q(p)). \quad (44)$$

Distributions ΔG_q and G_q^\pm measure number of quarks in the element d^3p . They can be, similarly as distribution G_q in Eq. (28), replaced by the distributions ΔP_q and P_q^\pm measuring number of quarks in the element dp/M :

$$\Delta P_q(p) = 3\Delta q(x) + 2 \int_x^1 \frac{\Delta q(y)}{y} dy - x\Delta q'(x); \quad x = \frac{2p}{M}, \quad (45)$$

$$P_q^\pm(p) = \frac{1}{2} \left[q(x) - xq'(x) \pm \left(3\Delta q(x) + 2 \int_x^1 \frac{\Delta q(y)}{y} dy - x\Delta q'(x) \right) \right]; \quad x = \frac{2p}{M}. \quad (46)$$

Obviously the probability distributions should satisfy

$$|\Delta G_q(p)| \leq G_q(p), \quad (47)$$

which after inserting from Eqs. (43) and (27) implies

$$\left| \left(\frac{\Delta q(x) + 2\Delta q_T(x)}{x} \right)' \right| \leq - \left(\frac{q(x)}{x} \right)', \quad (48)$$

where positivity of right hand side was required in relation (29). Another self-consistency test of the approach is represented by the inequality

$$|\Delta q(x)| \leq q(x), \quad (49)$$

which is proved in the Appendix C.

With the use of the relation (17) one can formally calculate the partial structure functions corresponding to the subsets of positively and negatively polarized quarks:

$$f_q^\pm(x) = Mx \int G_q^\pm(p) \delta \left(\frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}. \quad (50)$$

Apparently it holds

$$f_q(x) \equiv f_q^+(x) + f_q^-(x) = q(x) \quad (51)$$

and one can define also

$$\Delta f_q(x) = f_q^+(x) - f_q^-(x) \quad (52)$$

or equivalently

$$\Delta f_q(x) = Mx \int \Delta G_q(p) \delta \left(\frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}. \quad (53)$$

Obviously

$$f_q^\pm(x) = \frac{1}{2} (f_q(x) \pm \Delta f_q(x)) \quad (54)$$

and Eq. (47) implies

$$|\Delta f_q(x)| \leq q(x). \quad (55)$$

Let us note, $f_q^+ + f_q^- = q$, but $f_q^+ - f_q^- \neq \Delta q$ in the sense of the relations (17) and (18). The last inequality is replaced by equality only in the limit of IMF approach. The relation (53) can be written as

$$\Delta f_q(x) = xV_{q,-1}(x), \quad (56)$$

where

$$V_{q,-1}(x) = M \int \Delta G_q(p) \delta \left(\frac{p_0 + p_1}{M} - x \right) \frac{d^3 p}{p_0}. \quad (57)$$

At the same time Eq.(37) can be replaced by

$$V_{q,-1}(x) = \frac{1}{x} \left(\Delta q(x) + 2 \int_x^1 \frac{\Delta q(y)}{y} dy \right). \quad (58)$$

which after inserting from Eq. (56) gives

$$\Delta f_q(x) = \Delta q(x) + 2 \int_x^1 \frac{\Delta q(y)}{y} dy. \quad (59)$$

This equality together with the relation (55) give

$$\left| \Delta q(x) + 2 \int_x^1 \frac{\Delta q(y)}{y} dy \right| \leq q(x) \quad (60)$$

or equivalently

$$|\Delta q(x) + 2\Delta q_T(x)| \leq q(x). \quad (61)$$

Now, using the input on the $q(x)$ [25] and $\Delta q(x)$ [26] (LO at the scale $4GeV^2$) one can calculate the distributions $\Delta P_q, P_q, P_q^\pm$ and related structure functions $\Delta f_q, f_q$ and f_q^\pm . The result is displayed in Fig. 2 and one can observe:

i) Positivity of distributions P_q^\pm and f_q^\pm implies, that self-consistency tests (47), (55) and their equivalents (48), (60) are satisfied with exception of a small negative disturbance in $G_u^-(P_u^-)$ and f_u^- . The possible reason is that the results of the two different procedures for fitting $q(x)$ and $\Delta q(x)$ are combined and some uncertainty is unavoidable.

ii) The mean value of the distribution ΔG_q can be estimated as

$$\langle p \rangle_q = \frac{\int p \Delta G_q(p) d^3 p}{\int \Delta G_q(p) d^3 p} = \frac{M}{2} \frac{\int_0^1 x \Delta q(x) dx}{\int_0^1 \Delta q(x) dx}. \quad (62)$$

The proof of this relation is done in the Appendix D. The numerical calculation gives $0.090GeV/c$ for u and $0.070GeV/c$ for d quarks. These numbers are well comparable with those calculated from Eq. (35), which corresponds to the valence quarks. Also the shape of distributions $x\Delta f_u(x)$ and $x\Delta f_d(x)$ is very similar to that of the valence terms. In other words, results confirm that spin contribution of quarks comes dominantly from the valence region.

iii) Due to input values $\Delta u(x) > 0$ and $\Delta d(x) < 0$ one can expect that $P_u^+ \geq P_u^-$, $P_d^- \geq P_d^+$, $f_u^+ \geq f_u^-$ and $f_d^- \geq f_d^+$. Besides, the curves in the figure show, that P_u^-, P_d^+, f_u^- and f_d^+ are close to zero, at least in the valence region.

3. INTRINSIC QUARK MOTION AND ORBITAL MOMENTUM

The rule of quantum mechanics says, that angular momentum consists of the orbital and spin part $\mathbf{j} = \mathbf{l} + \mathbf{s}$ and that in the relativistic case the \mathbf{l} and \mathbf{s} are not conserved separately, but only the total angular momentum j is conserved. This simple fact was in the context of quarks inside the nucleon pointed out in [27]. It means, that only j^2, j_z are well-defined quantum numbers and corresponding states of the particle with spin 1/2 are represented by the bispinor spherical waves [28]

$$\psi_{kjlj_z}(\mathbf{p}) = \frac{\delta(p-k)}{p\sqrt{2p_0}} \begin{pmatrix} i^{-l} \sqrt{p_0 + m} \Omega_{jlj_z}(\omega) \\ i^{-\lambda} \sqrt{p_0 - m} \Omega_{j\lambda j_z}(\omega) \end{pmatrix}, \quad (63)$$

where $\omega = \mathbf{p}/p$, $l = j \pm \frac{1}{2}$, $\lambda = 2j - l$ (l defines the parity) and

$$\begin{aligned} \Omega_{j,l,j_z}(\omega) &= \begin{pmatrix} \sqrt{\frac{j+l_z}{2j}} Y_{l,j_z-1/2}(\omega) \\ \sqrt{\frac{j-l_z}{2j}} Y_{l,j_z+1/2}(\omega) \end{pmatrix}; \quad l = j - \frac{1}{2}, \\ \Omega_{j,l,j_z}(\omega) &= \begin{pmatrix} -\sqrt{\frac{j-l_z+1}{2j+2}} Y_{l,j_z-1/2}(\omega) \\ \sqrt{\frac{j+l_z+1}{2j+2}} Y_{l,j_z+1/2}(\omega) \end{pmatrix}; \quad l = j + \frac{1}{2}. \end{aligned}$$

States are normalized as:

$$\int \psi_{k'j'l'j'_z}^\dagger(\mathbf{p}) \psi_{klj_z}(\mathbf{p}) d^3p = \delta(k - k') \delta_{jj'} \delta_{ll'} \delta_{j_z j'_z}. \quad (64)$$

The wavefunction (63) is simplified for $j = j_z = 1/2$ and $l = 0$. Taking into account that

$$Y_{00} = \frac{1}{\sqrt{4\pi}}, \quad Y_{10} = i\sqrt{\frac{3}{4\pi}} \cos \theta, \quad Y_{11} = -i\sqrt{\frac{3}{8\pi}} \sin \theta \exp(i\varphi),$$

one gets:

$$\psi_{klj_z}(\mathbf{p}) = \frac{\delta(p - k)}{p\sqrt{8\pi p_0}} \begin{pmatrix} \sqrt{p_0 + m} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ -\sqrt{p_0 - m} \begin{pmatrix} \cos \theta \\ \sin \theta \exp(i\varphi) \end{pmatrix} \end{pmatrix}. \quad (65)$$

Let us note, that $j = 1/2$ is the minimum angular momentum for the particle with spin $1/2$. If one consider the quark state as a superposition

$$\Psi(\mathbf{p}) = \int a_k \psi_{klj_z}(\mathbf{p}) dk; \quad \int a_k^* a_k dk = 1 \quad (66)$$

then its average spin contribution to the total angular momentum reads:

$$\langle s \rangle = \int \Psi^\dagger(\mathbf{p}) \Sigma_z \Psi(\mathbf{p}) d^3p; \quad \Sigma_z = \frac{1}{2} \begin{pmatrix} \sigma_z & \\ & \sigma_z \end{pmatrix}. \quad (67)$$

After inserting from Eqs. (65), (66) into (67) one gets

$$\langle s \rangle = \int a_p^* a_p \frac{(p_0 + m) + (p_0 - m)(\cos^2 \theta - \sin^2 \theta)}{16\pi p^2 p_0} d^3p = \frac{1}{2} \int a_p^* a_p \left(\frac{1}{3} + \frac{2m}{3p_0} \right) dp. \quad (68)$$

Since $j = 1/2$, the last relation implies for the quark orbital momentum:

$$\langle l \rangle = \frac{1}{3} \int a_p^* a_p \left(1 - \frac{m}{p_0} \right) dp. \quad (69)$$

It means that for quarks in the state $j = j_z = 1/2$ there are the extreme scenarios:

i) Massive and static quarks ($p_0 = m$), which implies $\langle s \rangle = j = 1/2$ and $\langle l \rangle = 0$. This is evident, since without kinetic energy no orbital momentum can be generated.

ii) Massless quarks ($m \ll p_0$), which implies $\langle s \rangle = 1/6$ and $\langle l \rangle = 1/3$.

Generally, for $p_0 \geq m$, one gets $1/3 \leq \langle s \rangle / j \leq 1$. In other words, for the states with $p_0 > m$ part of the total angular momentum $j = 1/2$ is necessarily generated by the orbital momentum. This is a consequence of quantum mechanics, and not a consequence of particular model. If one assumes the quark effective mass of the order thousandths and intrinsic momentum of the order of tenth of GeV , which is quite realistic consideration, then the second scenario is clearly preferred. Further, the mean kinetic energy corresponding to the superposition (66) reads

$$\langle E_{kin} \rangle = \int a_p^* a_p E_{kin} dp; \quad E_{kin} = p_0 - m \quad (70)$$

and at the same time the Eq. (69) can be rewritten as

$$\langle l \rangle = \frac{1}{3} \int a_p^* a_p \frac{E_{kin}}{p_0} dp. \quad (71)$$

It is evident, that for fixed $j = 1/2$ both the quantities are in the nucleon rest frame almost equivalent: more kinetic energy generates more orbital momentum and vice versa.

Further, the average spin part $\langle s \rangle$ of the total angular momentum $j = 1/2$ related to single quark according to Eq. (68) can be compared to the integral

$$\Gamma_1 = \int_0^1 g_1(x) dx, \quad (72)$$

which measures total quark spin contribution to the nucleon spin. For the g_1 from Eq. (15) this integral reads

$$\Gamma_1 = \frac{1}{2} \int \Delta G(p_0) \left(\frac{1}{3} + \frac{2m}{3p_0} \right) d^3p. \quad (73)$$

Dependence of both the integrals (68) and (73) on intrinsic motion is controlled by the same term $(1/3 + 2m/3p_0)$, which in both the cases has origin in the covariant kinematics of the particle with $s = 1/2$. In fact, the procedures for calculation of these integrals are based on the two different representations of the solutions of Dirac equation: the plane waves (1) and spherical waves (65). It is apparent that for the scenario of massless quarks ($m \ll p_0$), due to necessary presence of the orbital motion, both the integrals Γ_1 and $\langle s \rangle$ will be roughly three times less, than for the scenario of massive and static quarks ($m \simeq p_0$). What is the underlying physics behind the interplay between the spin and orbital momentum? Actually, speaking about the spin of the particle represented by the state (1), one should take into account:

a) Definite projection of the spin in the direction \mathbf{n} is well-defined quantum number only for the particle at rest ($p = 0$) or for the particle moving in the the direction \mathbf{n} , i.e. $\mathbf{p}/p = \pm \mathbf{n}$. In these cases we have

$$s = u^\dagger(\mathbf{p}, \lambda \mathbf{n}) \mathbf{n} \Sigma u(\mathbf{p}, \lambda \mathbf{n}) = \pm 1/2. \quad (74)$$

b) In other cases, as shown in the Appendix E, only inequality

$$\langle s \rangle = |u^\dagger(\mathbf{p}, \lambda \mathbf{n}) \mathbf{n} \Sigma u(\mathbf{p}, \lambda \mathbf{n})| < 1/2 \quad (75)$$

is satisfied. Roughly speaking, the result of measuring the spin (of a quark) depends on its momentum in the defined reference frame (nucleon rest frame). This obvious effect acts also in the states, which are represented by the superposition of the plane waves (1) with different momenta \mathbf{p} and resulting in $\langle \mathbf{p} \rangle = 0$, but $\langle \mathbf{p}^2 \rangle > 0$. In [11] we showed, that averaging of the spin projection (75) over the spherical momentum distribution gives the result equivalent to (73). The state (66) can be also decomposed into plane waves having spherical momentum distribution and the spin mean value given by Eq. (68). Well-defined quantum numbers $j = j_z = 1/2$ imply, that the spin reduction due to increasing intrinsic kinetic energy is compensated by the increasing orbital momentum.

Now, what the preferred scenario of massless quarks ($\langle m/p_0 \rangle \ll 1$) implies for the spin structure of whole nucleon, what are the integral quark spin and orbital contributions to the nucleon spin? Obviously using some input on the total quark longitudinal polarization $\Delta\Sigma$, one can estimate the relative quark spin and orbital contributions as:

$$S = \Delta\Sigma, \quad L = 2\Delta\Sigma; \quad \Delta\Sigma = \sum_q \int_0^1 \Delta q(x) dx. \quad (76)$$

At the same time our approach can be compared with the calculation based on the chiral quark soliton model (CQSM) [21],[22], where significant role of the quark orbital momentum is considered as well. In Tab. I some results of both models are shown. However, in spite of some similarity between the two sets of numbers, there are substantial differences between both the approaches. Let us mention at least the two, which seem to be most evident:

1) Presence of significant fraction of the orbital momentum in the CQSM apparently follows from dynamics inherent in the model. On the other hand, in our approach the important role of the orbital momentum follows from kinematics, so it should not be too sensitive to details of inherent dynamics. Actually effect takes place in LO when quarks interacting with probing photon can be effectively described as free fermions in the states like (66) with sufficiently low effective ratio $\langle m/p_0 \rangle$ which controls the fraction of orbital momentum (69). Of course, value of this ratio itself is question of the dynamics.

2) In the CQSM antiquarks are predicted to have opposite signs of the spin and orbital contributions. In our approach both contributions are proportional and have the same signs regardless of flavor or antiflavor.

Last comment concerns the total quark angular momentum J , by which a room for the gluon contribution J_g is defined. Results in Tab. I related to the CQSM suggest, that higher Q^2 implies greater gluon contribution. Our results suggest, that gluon contribution can be rather sensitive to the longitudinal polarization: for $\Delta\Sigma \simeq 1/3, 0.3$ and 0.2 the gluon contribution can represent $\simeq 0, 10$ and 40% respectively. Let us remark, that the value empirically known [22]

$$\Delta\Sigma \simeq 0.2 - 0.35 \quad (77)$$

does not exclude any of these possibilities.

4. SUMMARY AND CONCLUSION

We studied covariant version of the QPM with spherically symmetric distributions of the quark momentum in the nucleon rest frame. The main results obtained in this paper can be summarized as follows.

1) Relations between the distribution functions $q(x)$, $\Delta q(x)$ and corresponding 3D quark momentum distributions $G_q^\pm(p) = G_q(p) \pm \Delta G_q(p)$ are obtained. In this way the momentum distributions of positively and negatively polarized quarks $G_q^\pm(p)$ are calculated from the input, which is obtained from experimentally measured structure functions F_2 and g_1 . At the same time these relations, due to positivity of probabilistic distributions G_q and G_q^\pm , imply some inequalities for $q(x)$, $\Delta q(x)$. We proved, that these constraints, serving as self-consistency tests of the approach, are satisfied.

2) We showed, that important role of the quark orbital momentum emerges as a direct consequence of a covariant description. Since in relativistic case only the total angular momentum $\mathbf{j} = \mathbf{l} + \mathbf{s}$ is well-defined quantum number, there arises some interplay between its spin and orbital parts. For the quark in the state with definite projection $j_z = 1/2$ in the nucleon rest frame, as a result of this interplay, its spin part is reduced in favor of the orbital one. The role of orbital motion increases with the rate of quark intrinsic motion; for $\langle m/p_0 \rangle \ll 1$ its fraction reaches $\langle l_z \rangle = 2/6$ whereas $\langle s_z \rangle = 1/6$ only. Simultaneously this effect is truly reproduced also in the formalism of structure functions and in this connection some implications for the global nucleon spin structure were suggested.

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APPENDIX A: STRUCTURE FUNCTIONS IN THE APPROACH OF INFINITE MOMENTUM FRAME

The necessary condition for obtaining equalities (19) - (20) is the covariant relation

$$p_\alpha = yP_\alpha, \quad (\text{A1})$$

which implies

$$m = yM \quad (\text{A2})$$

and $\mathbf{p} = 0$ in the nucleon rest frame and $p_T = 0$ in the IMF.

For calculation of the integrals (5) and (6) in the IMF approach one can substitute p by yP and d^3p/p_0 by $\pi dp_T^2 dy/y$. Then, after some algebra the structure functions (4) read

$$F_1(x) = \frac{1}{2} M x \int G(yM) \delta(y-x) \pi dp_T^2 \frac{dy}{y}, \quad F_2(x) = M x^2 \int G(yM) \delta(y-x) \pi dp_T^2 \frac{dy}{y}. \quad (\text{A3})$$

Since the approximation (A1) implies sharply peaked distribution at $p_T^2 \rightarrow 0$, one can identify

$$M G_q(yM) \pi dp_T^2 = q(y) \quad (\text{A4})$$

and then the Eqs. (19) and (A3) after integrating are equivalent.

In the same way the equalities (10)-(12) can be modified. Taking into account that $pS \rightarrow yPS = 0$, one obtain

$$g_1(x) = \frac{m}{2} \int \Delta G(yM) \delta(y-x) \pi dp_T^2 \frac{dy}{y}, \quad g_2(x) = 0. \quad (\text{A5})$$

If we put

$$M \Delta G_q(yM) \pi dp_T^2 = \Delta q(y) \quad (\text{A6})$$

and take into account Eq. (A2), then it is obvious, that the Eqs. (20) and (A5) are equivalent.

APPENDIX B: PROOF OF THE RELATION (37)

In the paper [12] we proved relation

$$\frac{V'_j(x)}{V'_k(x)} = \left(\frac{x}{2} + \frac{x_0^2}{2x} \right)^{j-k}; \quad x_0 = \frac{m}{M}, \quad (\text{B1})$$

which for $m \rightarrow 0$ implies

$$V_0(x) = \frac{1}{2} \left(xV_{-1}(x) + \int_0^x V_{-1}(y)dy \right). \quad (\text{B2})$$

After inserting V_0 from this relation to Eq. (36) one gets

$$\begin{aligned} g_1(x) = & \frac{1}{2} \left(xV_{-1}(x) + \int_0^x V_{-1}(y)dy \right) \\ & - 2x^2 \left(\int_x^1 \frac{V_{-1}(y)}{y^2} dy + \int_x^1 \frac{1}{y^3} \int_y^1 V_{-1}(z)dz dy \right) \\ & + \frac{1}{2}x \left(\int_x^1 \frac{V_{-1}(y)}{y} dy + \int_x^1 \frac{1}{y^2} \int_y^1 V_{-1}(z)dz dy \right). \end{aligned} \quad (\text{B3})$$

The double integrals can be reduced by integration by parts with the use of formula

$$\int_x^1 a(y) \left(\int_y^1 b(z)dz \right) dy = \int_x^1 (A(y) - A(x)) b(y)dy; \quad A'(x) = a(x), \quad (\text{B4})$$

then the relation (B3) is simplified:

$$g_1(x) = \frac{1}{2}xV_{-1}(x) - x^2 \int_x^1 \frac{V_{-1}(y)}{y^2} dy. \quad (\text{B5})$$

In the next step we extract V_{-1} from this relation. After the substitution $V(x) = V_{-1}(x)/x$ the relation reads

$$\frac{g_1(x)}{x^2} = \frac{1}{2}V(x) - \int_x^1 \frac{V(y)}{y} dy, \quad (\text{B6})$$

which implies the differential equation for $V(x)$:

$$\frac{1}{2}V'(x) + \frac{V(x)}{x} = \left(\frac{g_1(x)}{x^2} \right)'. \quad (\text{B7})$$

The corresponding homogeneous equation

$$\frac{1}{2}V'(x) + \frac{V(x)}{x} = 0 \quad (\text{B8})$$

gives the solution

$$V(x) = \frac{C}{x^2}, \quad (\text{B9})$$

which after inserting to Eq. (B7) gives

$$C'(x) = 2x^2 \left(\frac{g_1(x)}{x^2} \right)'. \quad (\text{B10})$$

After integration one easily gets the relation inverse to Eq. (B5):

$$V_{-1}(x) = \frac{2}{x} \left(g_1(x) + 2 \int_x^1 \frac{g_1(y)}{y} dy \right), \quad (\text{B11})$$

which coincides with Eq. (37).

APPENDIX C: PROOF OF THE RELATION (49)

Relations (17) and (18) imply that inequality (49) is satisfied if

$$p_0 + p_1 \geq \left| m + p_1 + \frac{p_1^2}{p_0 + m} \right| = \left| p_0 + p_1 - \frac{p_T^2}{p_0 + m} \right|. \quad (\text{C1})$$

There are two cases:

a) $p_0 + p_1 - p_T^2/(p_0 + m) \geq 0$, then instead of the relation (C1) one can write

$$p_0 + p_1 \geq p_0 + p_1 - \frac{p_T^2}{p_0 + m}, \quad (\text{C2})$$

which is always satisfied.

b) $p_0 + p_1 - p_T^2/(p_0 + m) < 0$, then the relation (C1) is equivalent to

$$p_0 + p_1 \geq -p_0 - p_1 + \frac{p_T^2}{p_0 + m} \Leftrightarrow 2(p_0 + p_1) \geq \frac{p_T^2}{p_0 + m}. \quad (\text{C3})$$

Since

$$\begin{aligned} 2p_0 \geq p_0 - p_1 &\Rightarrow 2(p_0 + m) \geq p_0 - p_1 \Rightarrow 2(p_0 + m)(p_0 + p_1) \geq (p_0 - p_1)(p_0 + p_1) \\ &\Rightarrow 2(p_0 + m)(p_0 + p_1) \geq p_T^2 \Rightarrow 2(p_0 + p_1) \geq \frac{p_T^2}{p_0 + m}, \end{aligned}$$

which means, that (C3) is always satisfied. In this way the relations (C1) and (49) are proved.

APPENDIX D: PROOF OF THE RELATION (62)

The relation (40) implies

$$\int \Delta G_q(p) d^3 p = \frac{1}{2} \int_0^1 \left(3\Delta q(x) + 2 \int_x^1 \frac{\Delta q(y)}{y} dy - x\Delta q'(x) \right) dx \quad (\text{D1})$$

and

$$\int p \Delta G(p) d^3 p = \frac{M}{4} \int_0^1 \left(3x\Delta q(x) + 2x \int_x^1 \frac{\Delta q(y)}{y} dy - x^2 \Delta q'(x) \right) dx; \quad x = \frac{2p}{M}. \quad (\text{D2})$$

If one denotes

$$\Gamma_1^q = \int_0^1 \Delta q(x) dx, \quad \Gamma_2^q = \int_0^1 x \Delta q(x) dx, \quad (\text{D3})$$

then integration by parts gives

$$\int_0^1 \int_x^1 \frac{\Delta q(y)}{y} dy dx = \Gamma_1^q, \quad \int_0^1 x \Delta q'(x) dx = -\Gamma_1^q \quad (\text{D4})$$

and

$$\int_0^1 2x \int_x^1 \frac{\Delta q(y)}{y} dy dx = \Gamma_2^q, \quad \int_0^1 x^2 \Delta q'(x) dx = -2\Gamma_2^q. \quad (\text{D5})$$

Now, one can easily express the ratio

$$\frac{\int p \Delta G(p) d^3 p}{\int \Delta G(p) d^3 p} = \frac{M}{2} \frac{\Gamma_2^q}{\Gamma_1^q}, \quad (\text{D6})$$

in this way the relation (62) is proved.

APPENDIX E: PROOF OF THE RELATION (75)

With the use of rule

$$\mathbf{p}\sigma \cdot \mathbf{n}\sigma + \mathbf{n}\sigma \cdot \mathbf{p}\sigma = 2\mathbf{p}\mathbf{n} \quad (\text{E1})$$

the term in Eq. (75) can be modified as

$$\begin{aligned} u^\dagger(\mathbf{p}, \lambda\mathbf{n}) \mathbf{n}\Sigma u(\mathbf{p}, \lambda\mathbf{n}) &= \frac{1}{2N} \phi_{\lambda\mathbf{n}}^\dagger \left(\mathbf{n}\sigma + \frac{\mathbf{p}\sigma \cdot \mathbf{n}\sigma \cdot \mathbf{p}\sigma}{(p_0 + m)^2} \right) \phi_{\lambda\mathbf{n}} \\ &= \frac{1}{2N} \phi_{\lambda\mathbf{n}}^\dagger \left(\mathbf{n}\sigma + \frac{\mathbf{p}\sigma \cdot (-\mathbf{p}\sigma \cdot \mathbf{n}\sigma + 2\mathbf{p}\mathbf{n})}{(p_0 + m)^2} \right) \phi_{\lambda\mathbf{n}} \\ &= \frac{1}{2N} \phi_{\lambda\mathbf{n}}^\dagger \left(\mathbf{n}\sigma \left(1 - \frac{\mathbf{p}^2}{(p_0 + m)^2} \right) + \frac{2\mathbf{p}\mathbf{n} \cdot \mathbf{p}\sigma}{(p_0 + m)^2} \right) \phi_{\lambda\mathbf{n}} \\ &= \frac{1}{2p_0} \phi_{\lambda\mathbf{n}}^\dagger \left(m \cdot \mathbf{n}\sigma + \frac{\mathbf{p}\mathbf{n} \cdot \mathbf{p}\sigma}{p_0 + m} \right) \phi_{\lambda\mathbf{n}}. \end{aligned} \quad (\text{E2})$$

Since

$$\left| \phi_{\lambda\mathbf{n}}^\dagger \mathbf{n}\sigma \phi_{\lambda\mathbf{n}} \right| = 1, \quad \left| \phi_{\lambda\mathbf{n}}^\dagger \mathbf{p}\sigma \phi_{\lambda\mathbf{n}} \right| \leq p, \quad \mathbf{p}\mathbf{n} = p \cos \alpha, \quad (\text{E3})$$

it follows

$$\left| u^\dagger(\mathbf{p}, \lambda\mathbf{n}) \mathbf{n}\Sigma u(\mathbf{p}, \lambda\mathbf{n}) \right| \leq \frac{1}{2p_0} \left(m + \frac{p^2}{p_0 + m} \right) = \frac{1}{2}. \quad (\text{E4})$$

Obviously

$$\left| u^\dagger(\mathbf{p}, \lambda\mathbf{n}) \mathbf{n}\Sigma u(\mathbf{p}, \lambda\mathbf{n}) \right| = \frac{1}{2} \quad (\text{E5})$$

only for $\mathbf{p}/p = \pm \mathbf{n}$ or $p = 0$.

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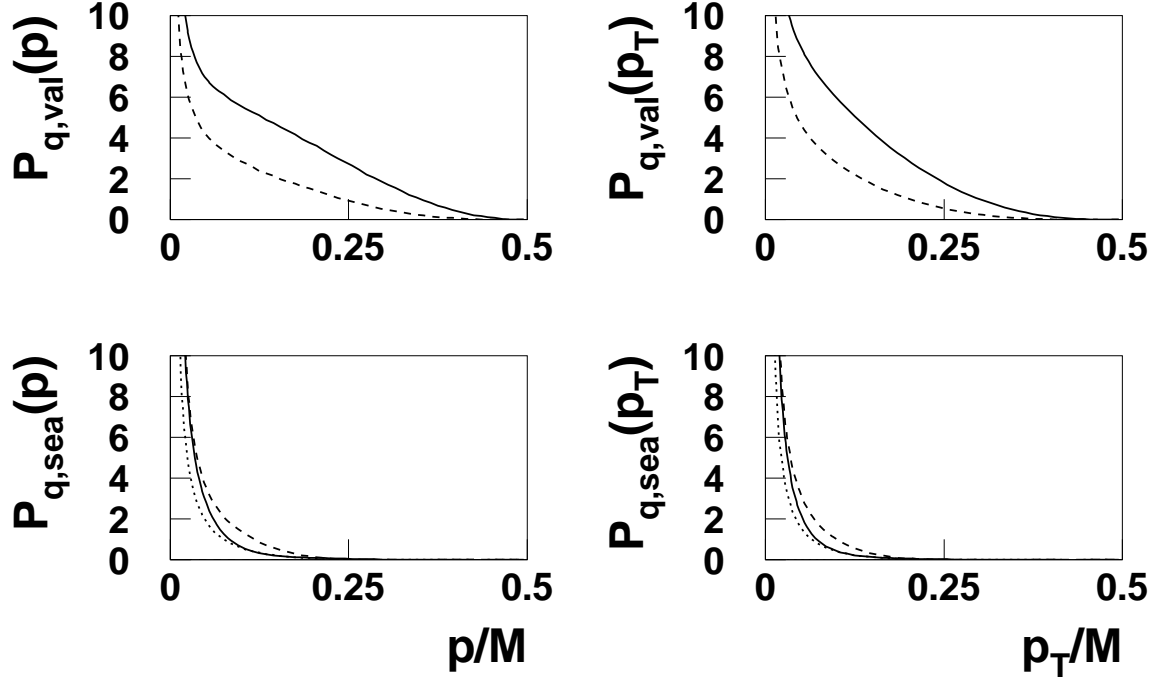


FIG. 1: The quark momentum distributions in the proton rest frame: the p and p_T distributions for valence quarks $P_{q,val} = P_q - P_{\bar{q}}$ and sea quarks $P_{\bar{q}}$ at $Q^2 = 4\text{GeV}^2$. Notation: u, \bar{u} - solid line, d, \bar{d} - dashed line, \bar{s} - dotted line.

CQSM			present paper	
	$Q^2=0.3\text{GeV}^2$	$Q^2=4\text{GeV}^2$	$\Delta\Sigma=0.2$	$\Delta\Sigma=0.3$
S	35.0	31.8	20.0	30.0
L	65.0	35.8	40.0	60.0
J	100.	67.6	60.0	90.0

TABLE I: Relative integral contributions of the quark spins (S), orbital momenta (L) and their sum (J) to the total nucleon spin. Results of our calculation (right) and prediction of the CQSM model (left).

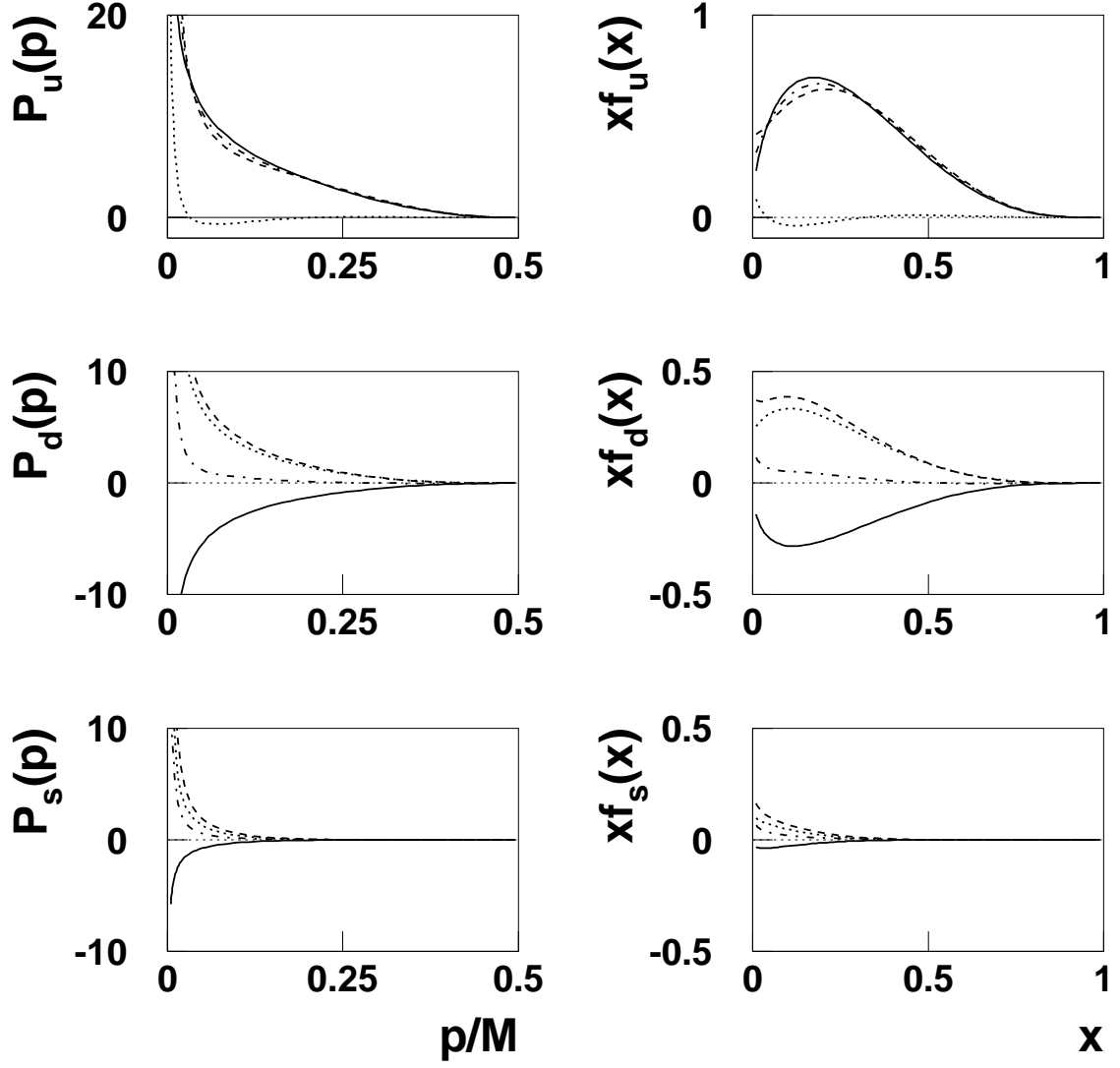


FIG. 2: Probability distributions $\Delta P_q, P_q, P_q^+, P_q^-$ of u, d, s quarks (left) and related structure functions $\Delta f_q, f_q, f_q^+, f_q^-$ (right) are represented by the solid, dashed, dash-and-dot and dotted lines.